

**MATHEMATICS DEPARTMENT**

**Year 12 MATHEMATICS SPECIALIST**

**TEST 2: VECTORS**

DATE: 3rd March 2016 Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Reading Time:** 3 minutes

**SECTION ONE: CALCULATOR FREE**

TOTAL: 25 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet.

WORKING TIME: 25 minutes (maximum)

**SECTION TWO: CALCULATOR ASSUMED**

TOTAL: 28 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 Calculators,

1 A4 page of notes (one side only), SCSA formula sheet.

WORKING TIME: 25 minutes (minimum)

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| --- | --- | --- | --- | --- | --- |
| **SECTION 1**  **Question** | **Marks available** | **Marks awarded** | **SECTION 2**  **Question** | **Marks available** | **Marks awarded** |
| **1** | **5** |  | **6** | **9** |  |
| **2** | **6** |  | **7** | **7** |  |
| **3** | **4** |  | **8** | **12** |  |
| **4** | **6** |  |  |  |  |
| **5** | **4** |  |  |  |  |
|  |  |  |  |  |  |
| **Total** | **25** |  |  | **28** |  |

Section One: Calculator-free [25 marks]

This section has five (5) questions. Answer all questions. Write your answers in the spaces provided

Question 1 [5 marks]

A straight line passes through the points  and .

(a) Find the vector equation of the line in the form . [2]

|  |
| --- |
| (1)  OR Suitable alternative (1) |

(b) Find the equation of the line through and  in parametric form. [1]

|  |
| --- |
| OR Suitable alternative (1) |

(c) Find the equation of the line through and  in Cartesian form. [2]

|  |
| --- |
| (1)  (1) |

Question 2 [6 marks]

The point A lies on the line with equation  and the point B has position

vector . Use a method involving a dot product to determine the position vector of A so

that the distance from A to B is a minimum. [6]

|  |
| --- |
| (1)  At point of closest approach  (1)  (1)  (1)    (1) (1) |

**Question 3 [4 marks]**

Point  has position vector  and point  has position vector . Find the

position vector of the point  that divides  internally in the ratio .

|  |
| --- |
| (1)  (1)    (1) (1) |

Question 4 [6 marks]

(a) Find a vector perpendicular to the two vectors:

 =  – 3 + 2

 = –2 +  –  [3]

|  |
| --- |
| (1) (1) (1) |

(b) If  and  are position vectors for the points  and , use your answer

to part (a), or otherwise, to find the area of the triangle . [3]

|  |
| --- |
| Area =  (1)  =  units2.  (1) (1) |

Question 5 [4 marks]

Points P and Q have coordinates  and  respectively.

(a) Write a vector equation for the line passing through  and . [2]

|  |
| --- |
| (1)  (1) |

(b) Show that the vector  is perpendicular to the line through  and . [1]

|  |
| --- |
| (1) |

(c) Write down a vector equation of the plane containing  and  with  as its normal vector. [1]

|  |
| --- |
| (1) |

**NAME:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Section Two: Calculator-assumed [25 marks]**

This section has three (3) questions. Answer all questions. Write your answers in the spaces provided

Question 6 [9 marks]

Two rockets are fired from different positions at the same time. Rocket 1 leaves from position  km at a velocity of  km/min and Rocket 2 leaves from position  km at a velocity of  km/min. Each rocket leaves a trail of smoke and, although the rockets do not collide, their smoke trails do intersect.

(a) Find the coordinates of the point at which the smoke trails intersect. [4]

|  |
| --- |
| Rocket 1:    Rocket 2:  At point of intersection:  and  (1)  (1)  This result for  and  gives the same z-component of . (1)  Thus, point of intersection is  (1) |

(b) Find the position of Rocket 1 three minutes after firing. [1]

|  |
| --- |
| For Rocket 1:  (1) |

(c) Find the shortest distance of Rocket 1 from the smoke trail of Rocket 2, three minutes after firing. Give your answer to the nearest metre. [4]

|  |
| --- |
| For Rocket 1 at ,  (1)  Using CAS,  km at   minutes.  (1) (1)  Thus, shortest distance is 6 574 m. (1)  (Can use dot product also, for same result) |

Question 7 [7 marks]

(a) The equation of a sphere is given by . Determine the vector equation of the sphere. [3]

|  |
| --- |
| (1) (1)  Equation of sphere is . (1) |

(b) Determine the position vector(s) of the points of intersection between the sphere and the line . [4]

At point of intersection:

|  |
| --- |
| (1)  (1)  (1)  Position vectors of points of intersection are:  and  (1) |

Question 8 [12 marks]

Let , be an equation of line .

The plane  has a normal vector  and passes through the point .

(a) Show that the point  lies on the line . [2]

|  |
| --- |
| (1)    lies on the line  (1) |

(b) Give the normal vector equation of the plane . [2]

|  |
| --- |
| (1)  Normal vector equation of P is  (1) |

(c) Find the shortest distance that plane  is from the origin. [2]

|  |
| --- |
| (1) (1) |

(d) Show that the line  meets the plane  at the point . [3]

|  |
| --- |
| At point of intersection,  (1)    (1)  (1) |

(e) Find the angle between the line  and the plane . (Give your answer correct to 1 decimal place.) [3]

|  |
| --- |
| Direction of L is  Direction of normal is  (1)  Angle between L and  is 31.80 (1)  Angle between L and P is 900 – 31.80 = 58.20 (1) |

END OF QUESTIONS